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USE OF ILL-INFORMED MULTIPLE ATTRIBUTE UTILITY THEORY FOR DECIDING WHICH OF TWO MANAGEMENT INFORMATION SYSTEMS TO PURCHASE

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MAY 1990

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U.S. ARMY
AVIATION
SYSTEMS COMMAND

DISCLAIMER

USE OF ILL-INFORMED MULTIPLE ATTRIBUTE UTILITY THEORY FOR DECIDING WHICH OF TWO MANAGEMENT INFORMATION SYSTEMS TO PURCHASE

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MAY 1990

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This report details an effort to address the issue of how to buy management information systems. It considers a variety of issues, including projected changes to the workforce, compatibility of information systems with those of other organizations, and obsolescence. Methodology includes R. K. Sarin's ill-informed Multiple Attribute Utility Theory, in which only ordinal information is needed regarding probabilities and utility values. Uses linear programming to derive estimates for utility values.

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FOREWORD

This report summarizes research done by the U.S. Army Aviation Systems Command with regard to selection of Automatic Data Processing Equipment (ADPE) Systems. It uses Multi Attribute Utility Theory (MAUT) with poorly understood utility functions and probabilities. Linear programming is used to estimate the scaling constants of the utility functions, while a heuristic is used to estimate the probabilities associated with states-of-nature.

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I. Introduction

A. Background

1. A problem that frequently confronts both private and public organizations is that of deciding which, if any, of numerous projects should be funded. The situation at AVSCOM is that of deciding which of two competing information systems should be purchased and implemented. In particular, we are faced with making a decision between purchasing numerous software packages for our existing "stand-alone" workstations or of purchasing a network solution. The former offers the advantage of being much less expensive. The latter, on the other hand, promises much more support for worker productivity, much greater interoperability with other computer resources, and much greater freeing up of professional talent from administrative burdens.

2. Of particular interest was the issue of how to deal with obsolescence; i.e., how to decide when to retire an old management information system. An approach often recommended for private sector organizations, capital budgeting using either the discounted cash flow or capital assets pricing model rationale, was considered to be inappropriate since:

(1) This public sector organization would not be allowed to invest pools of cash, thus the assumptions for the discounted cash flow model would not be satisfied.

(2) It is not possible to relate the project's risk to that of other risk assets; further, the concept of shareholder value is inapplicable to our organization. Thus, the requirements and assumptions for the capital assets pricing model would not be satisfied.

B. Outline of Work

1. Our analysis proceeds in two general phases: "exploratory" and "decision making". In the first, our objective is to properly formulate the problem. What are the alternative states-of-nature ? What are the relevant attributes to be considered ? What are the exogenous variables ? What are the decision variables ? What is a good model of the situation ? In the second phase, our objective is to combine the output from the first phase with applicable decision analysis techniques in order to suggest a course-of-action to management. That is, we attempt to answer the question "What can management do to affect a change in the future that will benefit the organization ?" The specific technique thought to be most appropriate here was Multiple Attribute Utility Theory (MAUT) with incomplete knowledge of probabilities and utility values (hereafter referred to as ill-defined MAUT, or simply MAUT).

2. An outline of our approach is as follows:

a. Exploratory Phase:

(1) Scenario planning to identify likely states-of-nature as well as to inform the selection of attributes.

(2) Cross Impact Analysis (CIA) to develop an ordinal ranking of probabilities associates with the states-of-nature.

b. Decision-Making Phase:

(1) Use of heuristics to estimate values for the probabilities for which ordinal rankings had been previously developed.

(2) Use of a screening procedure to estimate an upper bound and lower bound for the utility values associated with each course-of-action, attribute, and state-of-nature.

(3) Use of the estimated probabilities to calculate the expected values of utility values associated with each course-of-action and attribute.

(4) Use of linear programming to estimate values for the scaling constants and the upper and lower bounds associated with each course-of-action.

(5) Testing of courses-of-action for fathoming. The criteria used was that a course-of-action would be fathomed if the utility value associated with its upper bound was less than or equal to that associated with the lower bound of at least one other course-of-action. That is, under no circumstances could it be expected to return a greater utility value than the least returns associated with some other course-of-action.

(6) In the event that more than one course-of-action remained unfathomed (in our case, of course, there were only two courses-of-action to start with), we reassessed the utility values associated with some particular event. This led to a reappraisal of the upper and lower bounds of the utility functions for each course-of-action, state-of-nature, and attribute. We then repeated steps 'ii' through 'v' above, once more attempting to eliminate from consideration all but one course-of-action. In the event that more than one still remained unfathomed, we repeated this loop ('6', followed by '2' through '5', followed by '6', _et cetera_) until only one remained.

3. For a flowchart illustrating this methodology, the reader can refer to Appendix A.

C. Literature Review.

1. The methodology used here was taken from a number of sources. For the scenario planning of the exploratory phase, we use Wack (Wack, 1985), Mobasheri (Mobasheri _et al_, 1989), and Jain (Jain, 1985). For cross impact analysis, we use Jain (Jain, 1985). For a description of cross impact analysis in Government Agencies, with a special emphasis on the Intelligence Community, see Godson (Godson, 1989). While not critical to this study, an excellent account of the use of the entropy concept as a tool to determine how many alternative courses-of-action to delineate, see Starr and Greenwood (Starr and Greenwood, 1977). The core of our methodology for the decision-making

phase, a MAUT screening technique, was addressed by Sarin (Sarin, 1977). The heuristics used to assign probabilities to the states-of-nature for which ordinal rankings had been previously developed is discussed in Fishburn (Fishburn, 1966). Kenney and Raiffa (Keeney and Raiffa, 1976) provide a thorough grounding in decision analysis and utility theory for situations involving multiple objectives and uncertainty. An account of more advanced concepts, such as bilateral utility functions, is to be found in Farquahr (Farquahr, 1977).

2. The inputs to the scenario planning came from numerous periodicals, discussions with informed individuals, et cetera.

II. Exploratory Phase

A. Scenario Planning

1. Scenario planning has been found to be quite useful for dealing with situations characterized by a great degree of turbulence (Mobasheri et al, 1989). In such situations, traditional forecasting techniques tend to exhibit rather poor performance, since the structure of the situation is apt to change after the model has been formulated; i.e., historical data are a poor indicator of the future (Carbone and Makridakis, 1986). There are many approaches to scenario planning that use different orders of formalization. For example, Greenwood and Starr (Greenwood and Starr, 1977) discuss using the concept of entropy to determine how many alternatives to evaluate. Our use of the technique was rather informal; our objectives were to determine the future states-of-nature relevant to the situation and to make an informed choice regarding attributes to be evaluated (for the latter, see Appendix B). In order to achieve the former, a list of events that might be relevant to the situation was drawn up. The events were then combined into coherent, logically consistent states-of-nature.

B. Cross Impact Analysis (CIA)

1. Cross Impact Analysis (CIA) is a technique that assesses the impacts of events upon each other. It identifies groups of reinforcing and inhibiting events, and so allows us to rank order the likelihood of states-of-nature. Godson (Godson, 1989) describes the use of this technique in the Intelligence Community. Briefly, it is a tableau oriented methodology in which events that correspond to the rows of the tableau are judged as making events corresponding to the columns of the tableau more likely, less likely, or as not affecting their likelihood. For a detailed description of this methodology, see Jains (Jains, 1985). For our study, the output desired was a rank order of the likelihood of the previously identified states-of-nature (Appendix C).

III. Decision Making Phase

A. Fishburn's Heuristics

1. Our next step is to estimate the probability associated with each state-of-nature. Fishburn (Fishburn, 1967) addresses a heuristic for this task. Having already rank ordered the states-of-nature in the CIA, we next sort them in descending order in terms of likelihood. The estimated probability associated with each state-of-nature is defined as:

$$p_j = 2(N - j + 1) / (N(N + 1))$$

where

p_j is the estimated probability associated with the jth state-of-nature,

N is the total number of states-of-nature considered,

j is the rank of a specific state-of-nature in terms of likelihood.

2. The values calculated for p_j for each of our four states-of-nature are presented in Appendix D.

B. Sarin's Algorithm

1. One of the chief limitations of conventional MAUT is the required intensity of interaction with decision makers. Various approaches requiring only minimal knowledge of preferences have been attempted. The one that we use is attributed to R. K. Sarin (Sarin, 1977). Its initial requirements are modest; no knowledge of the utility value associated with a specific level of performance for each attribute is required (though some estimation of such information will be required for subsequent iterations), and only an ordinal ranking of the attributes is required to derive estimates of the scaling constants. For a flowchart detailing the logic of this technique, refer to Appendix E.

2. The chief limitation of this method is that it assumes that the situation under consideration can be adequately modeled with an additive utility function. This is equivalent to assuming preference and value independence. Fishburn (Fishburn, 1965) derived the necessary and sufficient conditions for representation by additive utility functions; to wit, that the desirability of any lottery over a solution space be dependent only upon the marginal probability distributions over the attributes and not upon the joint distributions. We can think of this as saying that interaction between the attributes is not significant. For example, if a situation can be adequately represented by an additive utility function, then a decision maker will be indifferent between two lotteries, one with a fifty fifty chance of attaining either the best or worst of two attributes, the other with a fifty-fifty chance of attaining the best of one attribute and the worst of another or the worst of the first attribute and the best of the second. As it was believed that such an assumption should not be taken for granted, some limited empirical testing was used to validate it for our situation.

3. Sarin's technique is a method of partial enumeration that attempts to fathom all but one course-of-action. The rationale used is that if the best possible return (in terms of utility values) associated with some course of action is less than or equal to the worst possible return associated with some other course-of-action, then the former should be fathomed.

4. The first step is to construct a 'Scores Table'. This is a tableau that lists the level of performance expected for each attribute with each course-of-action and state-of-nature. In our case, we had two possible courses-of-action and four states-of-nature for each of nine attributes, thus giving us 72 (9 X 4 X 2) elements. Our Scores Table is given in Appendix F. For a discussion of the cost estimate used in this table, see Appendix G.

5. The next step is to derive the additive utility function's lower and upper bounds at each value of the Scores Table. For the initial iteration of the algorithm, Sarin defines these values as:

$$f(x_{ij}^k) = \begin{cases} 1 \text{ if } x_{ij}^k = x_i^* \\ 0 \text{ otherwise} \end{cases}$$

and

$$g(x_{ij}^k) = \begin{cases} 0 \text{ if } x_{ij}^k = x_{i*} \\ 1 \text{ otherwise} \end{cases}$$

where:

f is the lower bound associated for x_{ij}^k,
g is the upper bound associated for x_{ij}^k,
x_{ij}^k is the level of performance associated with the ith attribute, jth state-of-nature, and kth course-of-action,
x_i^* is the maximum level of performance associated with the ith attribute, and
x_{i*} is the minimum level of performance associated with the ith attribute.

Appendix H lists the lower and upper bounds for each attribute, course-of-action, and state-of-nature.

6. Next, we use the estimated values of the probabilities associated with each state-of-nature to find expectations for the lower and upper bounds on the utility values associated with each attribute and course-of-action. This reduces our 9 X 2 X 4 X 2 = 144 element table to a 9 X 2 X 2 = 36 element table. These expected values are shown in Appendix I.

7. Having calculated the expectations of the lower and upper bounds for each attribute for each course-of-action, we next must derive estimates for the lower and upper bounds for the whole utility function as well as for that function's scaling constants, coefficients of the single attribute utility functions that indicate the relative worth of each attribute to the decision maker. That is, for example, how much cost

savings will be sacrificed to achieve more performance in networking [5] Sarin's technique estimates values of these constants for each course-of-action using mathematical programming. The rationale is that the lower bound for the whole utility function will be no less than that combination of the lower bounds of the individual attributes giving the least possible utility value without violating certain side constraints. Similar rationale is applied to the upper bound. Now, if this extremely pessimistic estimate for the lower bound for some course-of-action is found to be equal to or greater than an equally optimistic estimate of the upper bound for some other course-of-action, then the latter can be fathomed, or omitted from further consideration. The precise nature of the mathematical programming problem to be solved is determined by the nature of the utility function. In our case, since we have a completely additive utility function, we need solve a linear programming problem for each course-of-action. The sets of constraints for the problems are identical and are found by considering the ordinal ranking of the attributes, the requirement that the sum of the scaling constants equal unity, and the requirement that the value taken by each of these constants be equal to or greater than some small real value, in our case arbitrarily taken as .01. The rationale for this last requirement is that a less stringent requisite of non-negativity, which is what is generally required of the decision variables in mathematical programming, would allow some of the scaling constants to assume values of 0. This is equivalent to saying that no importance whatever was attached to the corresponding attributes. But the fact that they had been selected for evaluation in the first place by either the decision maker or by other inputs would seem to rule out this possibility. In recognition of the relatively great importance attached to obsolescence and cost considerations, these attributes were set to the most important and second most important in the side constraints. For a listing of the two initial linear programming problems, refer to Appendix J.

8. Proceeding, we next test to see if one of the two alternative courses-of-action can be fathomed. We use the decision rule to fathom course-of-action K iff

$$Z = A^G - B^K \geq 0$$

where

A^G is the lower bound associated with course-of-action G, and
B^K is the upper bound associated with course-of-action K.

For this iteration, the requirements of the decision rule were not satisfied, as the calculated value of Z is -0.39.

9. Sarin indicates that we should next evaluate the utility values associated with various levels of performance associated with the attributes. Using this new information, we then reevaluate the lower and upper bound table, take expectations, re-estimate the whole utility function lower and upper bounds and the scaling constants, then once more ascertain if one alternative course-of-action can be fathomed. Seven iterations were required to fathom a course-of-action. For a summary of the values of A_G, B_K, and Z, see Appendix K.

IV. Conclusions and Recommendations

A. The analysis done indicates that the preferred course-of-action is to acquire the proposed network solution. In recognition of the premium value that associated with obsolescence, the decision variable associated with this attribute was more heavily weighted than any other. Cost was weighted second most heavily.

B. Should the resources to implement the network solution prove unavailable, however, the software upgrade should be pursued immediately as a fallback position. The cost of doing nothing could very well be loss of compatibility with Program Management Offices, as the latter are presently upgrading to more advanced electronic spreadsheets that include such features as windowing, linking, and three dimensionality. Should this Directorate find itself in a position of inability to support the PM's, we could well find ourselves to be seen as an expensive and unnecessary appendage. Given the present financial austerity, such could prove a precarious position.

C. What is really being said is that service to the customer, while always important, becomes life critical during times of declining resources.

D. As a suggestion for future study, we suggest the development of a decision model able to handle choices in both the long and the short run. It is our contention that the model spelled out in this paper still provides and optimal choice in the long run. It does not, however, adequately consider the short run.

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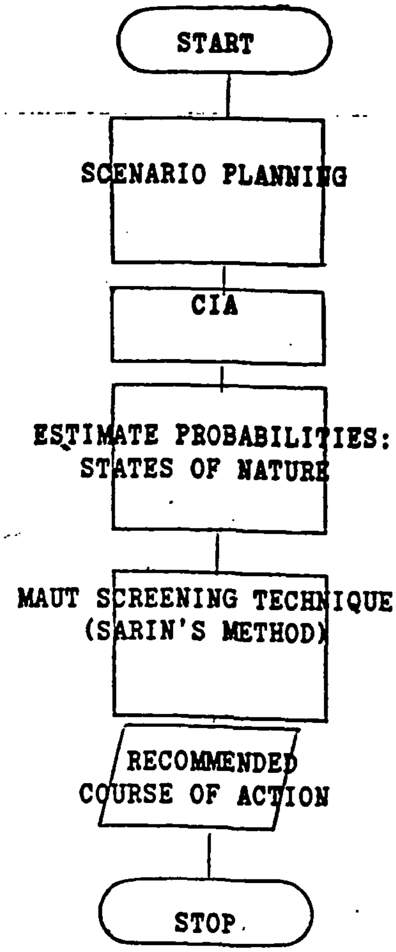
APPENDIX A
FLOWCHART OF METHODOLOGY



APPENDIX B
ATTRIBUTES & UNITS OF MEASURE

Attribute	Unit of Measure
Cost	Dollars
Networking Capability	0 - No; 1 - Yes
Software Installation Time	1 - Much; 5 - Little
Systems Backup Time	1 - Much; 5 - Little
File Sharing Time	1 - Much; 5 - Little
Obsolescence - Compatibility	1 - None; 5 - Total
Obsolescence - Useful Life Remaining	1 Year - 5 Years
Transparency to Users	1 None - 5 Total
Connectivity with OBCE	1 None - 5 Total

APPENDIX C
RANK ORDER OF STATES-OF-NATURE

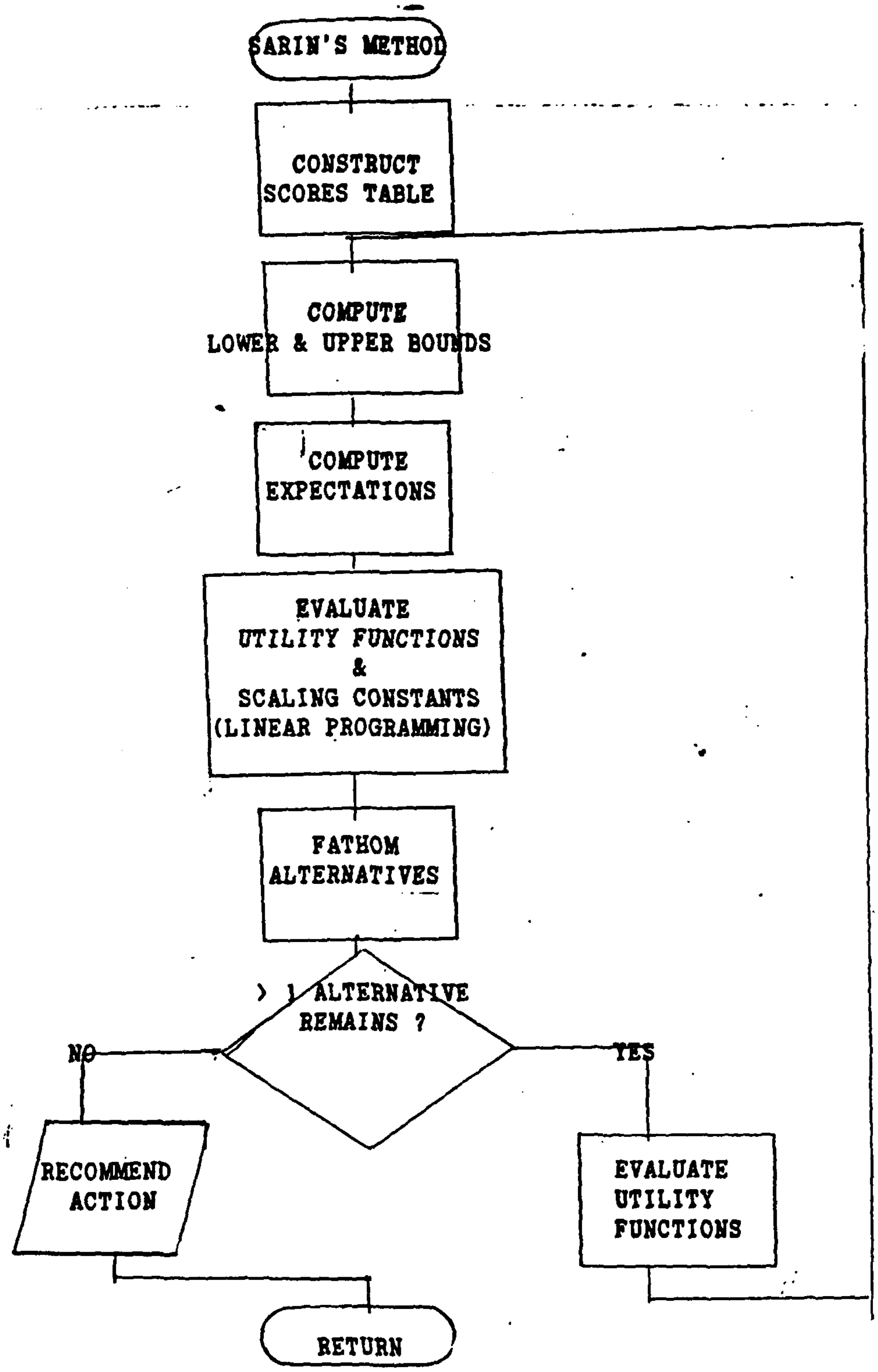
P_1 : Rapid Progress in Information Technologies & a RIF
P_2 : Rapid Progress in Information Technologies & NO RIF
P_3 : Moderate Progress in Information Technology & a RIF
P_4 : Moderate Progress in Information Technology & NO RIF

$P_2 > P_1 > P_4 > P_3$

APPENDIX D
PROBABILITIES OF STATES-OF-NATURE

State-of-Nature	Probability
P_1	.30
P_2	.40
P_3	.10
P_4	.20

APPENDIX E
FLOWCHART OF SARIN'S TECHNIQUE



APPENDIX F
SCORES TABLE

Course-of-Action	1				2			
State-of-Nature	1	2	3	4	1	2	3	4
Attribute								
1	5850				80,065			
2	0				1			
3	1	2	1	2	4	5	5	5
4	1	2	1	2	4	5	5	5
5	1	2	1	2	4	5	5	5
6	2	3	1	3	4	5	4	4
7	1	1	2	3	5	5	5	5
8	1	2	2	2	4	4	5	5
9	1	1	2	2	4	4	5	5

APPENDIX G
COST ESTIMATE

Course-of-Action #1: Upgrade Software

This alternative consists of purchasing a more advanced spreadsheet package, such as Lotus 1-2-3 Version 2.2 or Quatro Pro. By a special arrangement with the Lotus Development Corporation, the Directorate can trade-in its present electronic spreadsheets for Version 2.2 for $150 each. The total cost of upgrading the Directorate's 39 spreadsheet equipped workstations thus comes to $5,850. The cost of upgrading to Quatro Pro is $495 per package, or $19,305 for 39 workstations. The Lotus option is the alternative considered for this study. Discussions with relevant cost analysis personnel indicate, however, that the Lotus product lacks certain features essential to maintain compatibility with AVSCOM's Program Management Offices. Consequently, the upgrade to Quatro Pro would be necessitated. However, the cost of either of these options is so much less than that of the other course-of-action, the network solution proposed by Digitial Equipment Corporation, that there will be no effect upon the analysis.

Course-of-Action #2: Purchase Proposed Computer Network

This alternative consists of purchasing a network proposed by the Digital Equipment Corporation. This would include the following items:

1.	DEC VAXserver 3400 plus peripherals	$60,760
2.	DECNET Licence for 20 additional PC's	2,160
3.	Ethernet Boards for 20 additional PC's	11,820
4.	Lotus 1-2-3 Version 3.0 Network Version	1,593
5.	First Year Maintenance Cost, 3400	3,600
6.	First Year Maintenance, PC Boards	96
7.	First Year Maintenance, Monitors	36

Total Cost of Aquiring and Supporting System, First Year $80,065

This includes the cost of training, which would be provided by the Digital Equipment Corporation as part of the purchase price of the VAXserver 3400. This information was taken from Digital Equipment Corporation Quote 900AB0211-04.

APPENDIX H
LOWER & UPPER BOUNDS

BUTE		PROGRAM 1 S1	S2	S3	S4	PROGRAM 2 S1	S2	S3	S4
1	f	1	1	1	1	0	0	0	0
	g	1	1	1	1	0	0	0	0
2	f	0	0	0	0	1	1	1	1
	g	0	0	0	0	1	1	1	1
3	f	0	0	0	0	0	1	1	1
	g	0	1	0	1	1	1	1	1
4	f	0	0	0	0	0	1	1	1
	g	0	1	0	1	1	1	1	1
5	f	0	0	0	0	0	1	1	1
	g	0	1	0	1	1	1	1	1
6	f	0	0	0	0	0	1	0	0
	g	1	1	0	1	1	1	1	1
7	f	0	0	0	0	1	1	1	1
	g	0	0	1	1	1	1	1	1
8	f	0	0	0	0	0	0	1	1
	g	0	1	1	1	1	1	1	1
9	f	0	0	0	0	0	0	1	1
	g	0	0	1	1	1	1	1	1

APPENDIX I
EXPECTATIONS

BUTE		PROGRAMS 1	2
1	f	1	0
	g	1	0
2	f	0	1
	g	0	1
3	f	0	0.7
	g	0.3	1
4	f	0	0.7
	g	0.4	1
5	f	0	0.7
	g	0.4	1
6	f	0	0.3
	g	0.3	1
7	f	0	1
	g	0.3	1
8	f	0	0.3
	g	0.6	1
9	f	0	0.3
	g	0.3	1

APPENDIX J
INITIAL LINEAR PROGRAMMING TABLEAU

$$A^2 = \min z = w_2 + .7w_3 + .7w_4 + .7w_5 + .4w_6 + w_7 + .3w_8 + .3w_9$$

$$B^1 = \max z = w_1 + .6w_3 + .6w_4 + .6w_5 + .3w_6 + .3w_7 + .7w_8 + .3w_9$$

s.t.

$$w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_8 + w_9 = 1.0$$
$$w_6 + w_7 - w_1 > 0$$
$$w_3 + w_4 + w_5 - w_2 > 0$$
$$w_3 + w_4 + w_5 - w_8 - w_9 > 0$$
$$w_2 - w_8 + w_9 > 0$$
$$w_5 - w_4 > 0$$
$$w_4 - w_3 > 0$$
$$w_6 - w_7 > 0$$
$$w_1 - w_3 - w_4 - w_5 > 0$$
$$w_1 - w_2 > 0$$
$$w_1 - w_8 - w_9 > 0$$

$$w_i \geq .002,\ i = 1, 2, \ldots, 9.$$

Subsequent iterations of Sarin's algorithm yield, for the objective *function coefficients*, the following:

Iteration		weight coefficient 1	2	3	4	5	6	7	8	9
2	A	0	1	.7	.7	.7	.4	1	.3	.3
	B	.6	0	.6	.6	.6	.3	.3	.7	.3
3	A	0	1	.7	.7	.7	.4	1	.44	.3
	B	.6	0	.6	.6	.6	.3	.3	.14	.3
4	A	0	1	.7	.7	.7	.4	1	.44	.3
	B	.4	0	.6	.6	.6	.3	.3	.14	.3
5	A	0	1	.7	.7	.7	.4	1	.44	.3
	B	.4	0	.12	.12	.12	.3	.3	.14	.3
6	A	0	.7	0	.7	.7	.52	1	.44	.3
	B	.4	0	.12	.12	.12	.06	.3	.14	.3
7	A	0	1	.7	.7	.7	.88	1	.44	.3
	B	.4	0	.12	.12	.12	.18	.3	.14	.3

APPENDIX K
A_o, B_x, Z

Iteration	A_o	B_x	Z
1	.2390	.6325	-.3935
2	.2390	.4900	-.2510
3	.2404	.4844	-.2440
4	.2404	.4200	-.1796
5	.2404	.3335	-.0931
6	.2777	.2950	-.0173
7	.4588	.3056	.1532